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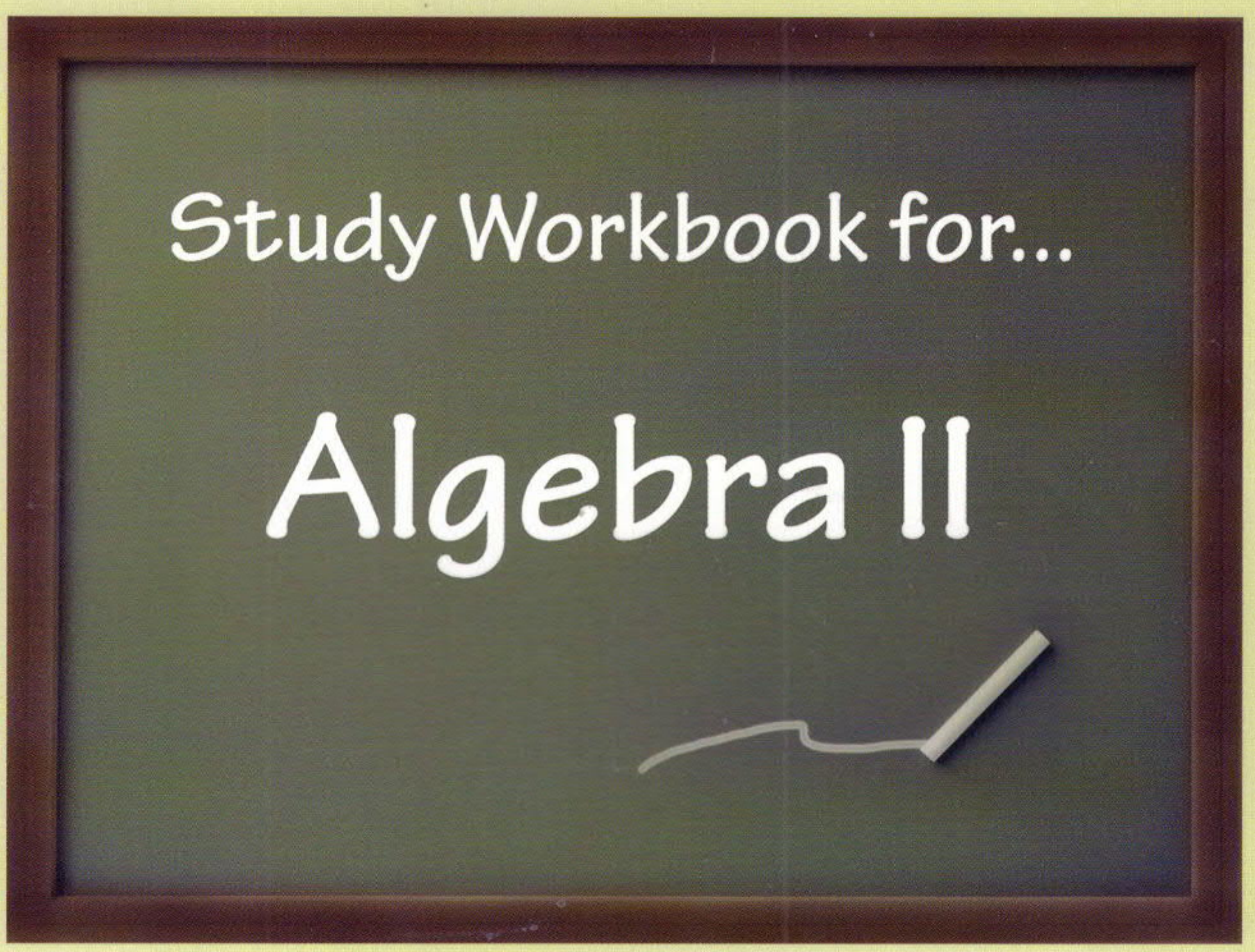

## Study Workbook for...

# Algebra II

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***Note: Lesson 1 is introductory. Questions and answers are not provided***

# *QUESTIONS*

# Lesson 2
# Polynomial Arithmetic

Solve the following:

**1.** $(x^2 - 3x + 7) + (x^3 - 2x^2 - 5x - 3)$

**2.** $(x^4 + 4x^3 - 6x + 1) - (3x^3 - 5x^2 + 4x + 1)$

**3.** $(5x^3 + 7x^2 - x - 3) + (2x^2 - 9x - 11) - (x^3 - 6x)$

**4.** $(3x + 7)(4x + 5)$

**5.** $(2x - 9)(2x - 1)$

**6.** $(5x + 1)(3x - 8)$

**7.** $(x^2 - 1)(x^2 + 3x - 1)$

**8.** $(x + 2)(2x - 3) - (3x - 2)(2x + 5)$

**9.** $4x(3x^2 - 2x + 7) - (2x - 1)^2$

**10.** $(3x + 2)(2x - 5)(x + 7)$

# Lesson 3
# *Factoring*

Factor each completely.

**1.** $8x^3 - 1$

**2.** $6x^2 - x - 35$

**3.** $5x^2 - 26x + 5$

**4.** $3x^6 - 192$

**5.** $x^3 - 4x^2 + 3x - 12$

**6.** $3x^3 + 3x^2 - 12x - 12$

Reduce each fraction by factoring and canceling common factors.

**7.** $\dfrac{x^2 - 16}{x^2 + 5x + 4}$

**8.** $\dfrac{x^4 - x}{x^4 + x^3 + x^2}$

**9.** $\dfrac{x^4 - 16}{x^4 - x^2 - 20}$

**10.** $\dfrac{x^2 + 3x - 40}{x^2 - 3x - 10}$

# Lesson 4

# Solving Linear Equations

Solve for $x$.

**1.** $x+3=4x-9$

**2.** $3x+7-7x=9-2x+8$

**3.** $\frac{1}{2}x+6=3$

**4.** $\frac{x+2}{5}=3$

**5.** $3(2x-5)=2(3-x)+3$

**6.** $x^2+3x-7=x^2+2x-1$

**7.** $x^2-4=x^2+3x+2$

**8.** $(x+5)(x-3)=(x+3)(x-2)$

**9.** $(x+7)(x-7)=(x+5)(x-4)$

**10.** $(x+3)(x-2)=(x+4)(x-1)$

# Lesson 5

## Solving Linear Inequalities

Solve the following:

**1.** $2x + 3 \geq 10$

**2.** $3x - 5 < x + 7$

**3.** $2x - 7 \leq 5x + 2$

**4.** $2x - 3 > 15$ or $3x - 7 < 2x$

**5.** $-5 \leq 3x + 1 \leq 19$

**6.** $|2x + 3| > 7$

**7.** $|5x - 10| \leq 5$

**8.** $|3 - 2x| < 5$

**9.** $|3x - 4| \leq x$

**10.** $|x^2+3x-12|<-2$

# Lesson 6
## Correlation, Slope, and Intercepts

We are trying to predict a student's grade on an Algebra II exam based on the number of minutes of studying time the night before the exam.

**1.** The slope is .3; what does this mean?

**2.** The $y$-intercept is 23; what does that mean?

**3.** Write the predicting equation.

**4.** What is the meaning of the $x$-intercept?

**5.** If the slope had been –2, what would that mean?

**6.** What causes a point to be above the line?

**7.** What might have caused the result in problem 6?

# Lesson 7

# The Equation of a Line

For each pair of equations, indicate whether the pair is parallel, perpendicular, or neither.

**1.** $y = 3x + 7$ $\quad$ $y = 7 - 3x$

**2.** $y = 2x + 5$ $\quad$ $3y - 6x = 11$

**3.** $y + x = 3$ $\quad$ $x + 7 = 10 - y$

**4.** $3x + y = 5$ $\quad$ $3y - x = -2$

Find the equation for each line.

**5.** Slope = 3 and (2,–5) is on the line.

**6.** Line contains (5,2) and (–3, 18).

**7.** Line contains (4,6) and (8,8).

**8.** Line contains (5,11) and is parallel to $y = -x + 17$.

**9.** Line contains (–9,2) and is perpendicular to $y = \frac{3}{2}x$.

**10.** Line contains (0,7)(–3,0).

# Lesson 8
## Graphing Linear Equations

For each problem, plot the two points, draw the line containing the points, find the slope and $y$-intercept from your graph, and write the equation of the line $y = mx + b$.

**1.** (2,3) (5,6)

**2.** (–2,7) (4,1)

**3.** (–3,5)(1,5)

**4.** (2,8) (–2,–8)

**5.** (–5,6) (–2,–3)

Graph each equation and use the line to find the $x$- and $y$-intercepts.

**6.** $y = 5x - 10$

**7.** $y = -3x - 6$

**8.** $y = 2x + 4$

**9.** $y = -x + 7$

**10.** $y - 3 = \frac{3}{2}(x - 4)$

# *Lesson 9*

## *Graphs of Linear Inequalities*

Graph the inequality $y > 2x - 6$. Determine from your graph whether each given point is a solution (in the shaded region).

**1.** 3,7

**2.** 5,2

**3.** –2,–10

**4.** –4,–1

Graph the inequality $x \leq 2$. Determine from your graph whether each given point is a solution.

**5.** –3,–7

**6.** 5,–1

**7.** 2,6

Graph the inequality $y > -1$. Determine from your graph whether each given point is a solution.

**8.** –2,6

**9.** –2,–2

**10.** 0,0

# Lesson 10
## Solving Systems of Two Linear Equations

Solve each system by graphing.

**1.** $x + y = 6$ $\quad$ $x - y = -2$

**2.** $y = 2x - 1$ $\quad$ $y = 3x + 4$

**3.** $y = 3x - 4$ $\quad$ $y = 1 - 2x$

Solve each system by substitution.

**4.** $x + 2y = 5$ $\quad$ $2x - 3y = -4$

**5.** $y = 3x - 7$ $\quad$ $2x - 3y = 0$

**6.** $x = 2 - y$ $\quad$ $3x - 2y = 21$

**7.** $3x = 6y + 9$ $\quad$ $2y - x = -3$

Solve each system by addition.

**8.** $x + y = 7$ $\quad$ $2x - y = 2$

**9.** $2x + 3y = -6$
$3x - 3y = -4$

**10.** $4x - 5y = 7$
$3y - 4x = -3$

# Lesson 11
## Solving Systems of Two Linear Equations Using Elimination

Solve each system.

**1.** $x + y = 9$
$\underline{x + 2y = 8}$

**2.** $3x - 2y = -2$
$\underline{x + 3y = -8}$

**3.** $4x + y = 8$
$\underline{y - 3x = 1}$

**4.** $2x + y = 0$
$\underline{3x - 2y = -14}$

**5.** $2x + 5y = 1$
$\underline{x - 2y = 5}$

**6.** $4x + 11 = 3y$
$\underline{2y + 4 = -3x}$

# Lesson 12

## Solving Systems of Three Linear Equations and Systems of Linear Inequalities

Solve each system.

**1.** $2x + y - z = 3$

$3x - y + 2z = -9$

$x - 2y - 3z = -6$

**2.** $3x + 2y - z = -3$

$2x - 3y + 2z = 20$

$3x + y - 4z = -16$

**3.** $x + 3y - 2z = -5$

$x + 3z = 16$

$x = 7$

**4.** $3x + 2y - z = -10$

$4x - 5y + 2z = -10$

$2x - 3y - 3z = -9$

**5.** $2x - 3y + 2z = 3$

$-2x - 5y + 3z = -15$

$-4x + 3y - 5z = 1$

Graph the system:

$y < 2x + 6$

$y \leq -3x + 6$

$y > 0$

From your graph, decide whether each given point is a solution.

**6.** (1,3)

**7.** (–2,3)

**8.** (–1,–1)

**9.** (3,5)

**10.** (0,4)

# Lesson 13
## Functions

Determine whether each relation is a function for the given domain.

**1.** $\{(3,2), (-3,2), (7,0), (0,7), (2,7)\}$

**2.** $y = 3x + 2$ for $x < -1$

**3.** $y = \frac{1}{2x}$ for $x > -5$

**4.** $y = \sqrt{x}$ for $-3 \leq x < 3$

**5.** $y = \frac{x+3}{x-3}$ for $x > 3$

**6.** $y = x^2 - \frac{1}{2}x$ for all $x$

Find the inverse for each function.

**7.** $f(x) = 3x + 2$

**8.** $f(x) = x^2 - 7$

**9.** $f(x) = \frac{1}{2x}$

**10.** $f(x) = \sqrt{x^2 + 1}$

# Lesson 14
# Quadratic Functions

For $y = x^2 - 6x - 16$:

1. Identify the vertex.

2. Identify the $y$-intercept.

3. Identify the point symmetric to the $y$-intercept.

4. Sketch the graph and identify the $x$-intercepts.

5. Is $(-3,-11)$ a point in your parabola?

For $y > x^2 + 2x - 15$, sketch the graph and answer the following:

6. Is $(-1,-16)$ a solution?

7. Is $(5,2)$ a solution?

8. Is $(-4,-6)$ a solution?

9. Is $(2,-5)$ a solution?

For $y = -x^2 + 6x - 5$:

**10.** Identify the vertex.

**11.** Identify the $y$-intercept.

**12.** Sketch the graph and identify the $x$-intercepts.

**13.** Is the origin above or below the vertex?

# Lesson 15
## Solving Quadratic Functions

Solve by factoring.

1. $x^2 + 7x - 18 = 0$

2. $x^2 - 9x + 20 = 0$

3. $2x^2 - 18 = 0$

4. $x^2 + 3x + 15 = 2x^2 - 13$

Solve by completing the sequence.

5. $x^2 + 6x - 2 = 5$

6. $2x^2 - 16x + 4 = 0$

Solve.

7. $x^2 - 12 = 0$

8. $x^2 + 9 = 9$

9. $x^2 - 5 = -3$

10. $x^2 - x = 16 - x$

# Lesson 16
# The Quadratic Formula

Solve using the quadratic formula.

1. $x^2-6x-72=0$

2. $2x^2+3x-4=0$

3. $3x^2+18x+27=0$

4. $x^2-x-20=0$

5. $x^2-3x+10=0$

For $y=2x^2+13x-7$:

6. What is the vertex?

7. What is the $y$-intercept?

8. What is the point that is symmetric to the $y$-intercept?

9. Graph the parabola and identify where the graph crosses the $x$-axis.

10. Check your answer to problem 9 by solving $2x^2+13x-7=0$.

# Lesson 17
## Imaginary Numbers

Solve the following:

**1.** $(4 + 7i) + (3 - 9i) - (5 - 6i) =$

**2.** $(3 - 6i)(2 + 4i) =$

**3.** $(5 - 2i)(3 - i) =$

**4.** Multiply $8 - 3i$ by its conjugate.

**5.** $(2 + 3i) \div (3 + 4i) =$

**6.** $3 + 7i + (5 - i)(2 + 3i) =$

Write each ordered pair as the complex number it represents.

**7.** $(-3,5)$

**8.** $(6,0)$

**9.** $\left(7,-\frac{2}{3}\right)$

Solve.

**10.** $x^2 - 2x + 17 = 0$

**11.** $x^2 + 3x + 10 = 0$

**12.** If $8-\frac{2}{3}i$ is a solution to a quadratic equation, what is the other solution?

# Lesson 18
## Quadratic and Rational Inequalities

Solve.

1. $x^2 + x - 12 > 0$

2. $x^2 - 6x + 8 \geq 0$

3. $x^2 + 9x + 20 > 0$

4. $x^2 + 5x - 6 \leq 0$

5. $x^2 - 7x + 12 < 0$

6. $x^2 - 3x - 40 < 0$

7. $\dfrac{x-3}{x+4} > 0$

8. $\dfrac{x-7}{2x-4} \geq 0$

9. $\dfrac{3x-5}{4-x} \leq 0$

10. $\dfrac{2x+3}{x+9} < 0$

# Lesson 19
## Polynomial Division

Divide using the synthetic method.

**1.** $x^3 + 3x^2 - 2x + 4 \div x + 1$

**2.** $x^4 - 3x^2 + 6x - 16 \div x - 2$

**3.** $x^4 - 3x^2 + 6x - 16 \div x^2 - 4$

**4.** $x^3 - 6x^2 + 80 \div x + 3$

**5.** $x^4 - 16 \div x - 2$

**6.** $x^3 - x \div x + 2$

**7.** Is $x + 1$ a factor of $x^3 + 3x^2 - 2x + 4$?

**8.** Is $x - 2$ a factor of $x^4 - 3x^2 + 6x - 16$?

**9.** Find a factor of $x^4 - 16$ using a problem already done in this lesson.

**10.** $x^2 + 4x + 6 \div 2x + 5$

# *Lesson 20*
# *Zeros of a Polynomial*

Solve the following:

**1.** List the possible rational zeros for $x^3 - 4x^2 + 4$.

**2.** List the possible rational zeros for $3x^5 + 4x^2 - 9x + 10$.

**3.** Find the zeros of $x^3 + 1$.

**4.** Find the zeros of $x^4 - x^3 - 7x^2 + 5x + 10$.

**5.** Find the zeros of $x^3 - 6x^2 + 9x$.

**6.** Find the zeros of $x^3 - 5x^2 + x - 5$.

**7.** Use the results from problem 3 to identify the $x$-intercept of $y = x^3 + 1$.

**8.** Use the results from problem 4 to find the factors of $x^4 - x^3 - 7x^2 + 5x + 10$.

**9.** Use the results from problem 5 to find the $x$-intercept of $y = x^3 - 6x^2 + 9x$.

**10.** Use the results from problem 6 to find the factors of $x^3 - 5x^2 + x - 5$.

# Lesson 21

# *Sketching Polynomials*

For $y = x^3 - 5x^2 + x - 5$:

**1.** Identify the $y$-intercept.

**2.** Identify the $x$-intercept.

**3.** Identify the end behavior.

**4.** How many turning points are possible?

**5.** Sketch the graph.

For $y = -x^3 + 6x^2 - 9x$:

**6.** Identify the $y$-intercept.

**7.** Identify the $x$-intercept.

**8.** Identify the end behavior.

**9.** How many turning points are possible?

10. Sketch the graph.

# Lesson 22
## Sketching Rational Functions

For $y = \dfrac{x}{x-2}$:

**1.** Identify the $y$-intercept.

**2.** Identify the $x$-intercept.

**3.** Identify the vertical asymptote.

**4.** Identify the horizontal asymptote.

**5.** Sketch the graph.

For $y = \dfrac{x-5}{x^2-1}$:

**6.** Identify the $y$-intercept.

**7.** Identify the $x$-intercept.

**8.** Identify the vertical asymptotes.

**9.** Identify the horizontal asymptote.

**10.** Sketch the graph.

# Lesson 23

## Square Roots and Cube Roots

**1.** Solve $\sqrt{2x+3}=x$.

**2.** Solve $\sqrt{x+7}=4$.

**3.** Solve $\sqrt{3x-1}=3$.

**4.** Solve $\sqrt[3]{x+4}=2$.

**5.** Solve $\sqrt[3]{x^2+2x}=x$.

For $y=\sqrt{x+4}-3$:

**6.** Identify the $y$-intercept.

**7.** Identify the $x$-intercept.

**8.** Identify the end point.

**9.** Sketch the graph.

For $y=\sqrt[3]{x-8}+1$:

**10.** Identify the $y$-intercept.

**11.** Identify the $x$-intercept.

**12.** Identify the "zero point."

**13.** Sketch the graph.

# Lesson 24
## Exponential Functions

Simplify each expression.

**1.** $3^x \bullet 3^{x-2}$

**2.** $x^y \div x^6$

**3.** $(x^2)^y$

**4.** $(3.14159)^{\circ}$

**5.** $\left(\frac{2}{3}\right)^{-2}$

**6.** Complete the roster for $y = 3^x$.

| $x$ | –2 | –1 | 0 | 1 | 2 | 4 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**7.** Complete the roster for $y = 5^{-x}$.

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

Solve:

**8.** $9^x = 27$

**9.** $8^x = \frac{1}{4}$

**10.** $4^{x+1} = 32^{x}$

# Lesson 25

# Logarithmic Functions

Write each logarithmic equation as an exponential equation and solve for $x$.

**1.** $\log_x 36 = 2$

**2.** $\log_5 x = -3$

**3.** $\log_7 343 = x$

**4.** $\log_x 16 = \frac{4}{3}$

**5.** $\log 5 + \log x = 2$

**6.** $\log_4 (x+13) - \log_4 (x-2) = 2$

**7.** Use a calculator to find $\log_7 19$.

**8.** What is the $\log_{\frac{1}{2}} 8$?

**9.** Can you find the $\log(-2)$?

**10.** Complete the roster for $y = \log_4 x$.

| $x$ | $\frac{1}{4}$ | $\frac{1}{2}$ | **1** | **2** | **4** | **8** | **16** |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

# Lesson 26
## Matrices and Determinants

Solve the following:

1. $\begin{bmatrix} 3 & -2 & 7 \\ 5 & 0 & -4 \end{bmatrix} - 2\begin{bmatrix} 6 & -3 & 1 \\ -2 & 5 & 3 \end{bmatrix} =$

2. $\begin{bmatrix} 2 & 3 & 1 \\ -1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 7 & 1 \\ -2 & 6 \end{bmatrix} =$

3. $\begin{bmatrix} 2 & -3 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 3 & -2 & -1 \end{bmatrix} =$

4. $\begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix}^2 =$

5. $\begin{vmatrix} 4 & 3 \\ -2 & -1 \end{vmatrix} =$

6. $\begin{vmatrix} 2 & 0 & 3 \\ 1 & -1 & 2 \\ 3 & -3 & 1 \end{vmatrix} =$

7. $\begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix}^{-1} =$

8. Verify your answer to problem 7.

9. $\begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}^{-1} =$

**10.** $\left(\begin{bmatrix} 2 & 6 \\ -3 & 5 \end{bmatrix}^2 - 2\begin{bmatrix} 4 & -7 \\ 3 & 2 \end{bmatrix}\right)^{-1} =$

# Lesson 27

## Solving Systems of Equations Using Matrices

Use the inverse matrix method to solve each system.

**1.** $x + y = 6$

$x - y = -2$

**2.** $2x - y = 1$

$3x - y = -4$

**3.** $3x - y = 4$

$2x + y = 1$

**4.** $x + 2y = 5$

$2x - 3y = -4$

**5.** $3x - y = 7$

$2x - 3y = 0$

**6.** $x + y = 2$

$3x - 2y = 21$

**7.** $x + y = 7$

$2x - y = 2$

**8.** $4x - 5y = 7$

$4x - 3y = 3$

Use Cramer's rule to solve:

**9.**

$$\begin{aligned} 3x + 2y - z &= -3 \\ 2x - 3y + 2z &= 20 \\ 3x + y - 4z &= -16 \end{aligned}$$

**10.**

$$\begin{aligned} 3x + 2y - z &= -10 \\ 4x - 5y + 2z &= -10 \\ 2x - 3y - 3z &= -9 \end{aligned}$$

# Lesson 28
# *Recursive Functions*

Complete the roster of points.

**1.** If $f(n) = 2f(n-1) + 3$:

| $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(n)$ | 5 | | | | |

**2.** If $f(n) = f(n-2) + f(n-1) - 1$:

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(n)$ | 1 | 2 | | | | |

**3.** If $f(n) = f(n-1) + 2$:

| $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(n)$ | 7 | | | | |

**4.** Verify that problem 3 is a linear function.

**5.** What is the slope of the linear function?

**6.** What is the equation of the line?

**7.** Complete the roster if $f(n) = f(n-1) + 2n$:

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(n)$ | 4 | | | | |

**8.** Is this a linear function?

**9.** Is this a quadratic function?

**10.** Find the quadratic function.

# *Lesson 29*
## *Sequences and Series*

**1.** Find $\sum_{i=1}^{4} i^2$.

For each sequence, determine if it is arithmetic, geometric, or neither.

**2.** 7, 9, 11, 13

**3.** 7, 14, 28, 56

**4.** 7, 9, 13, 19

**5.** Find the 50th term in the sequence in problem 2.

**6.** Find the sum of the first 20 terms of the sequence in problem 2.

**7.** Find the 10th term of the sequence in problem 3.

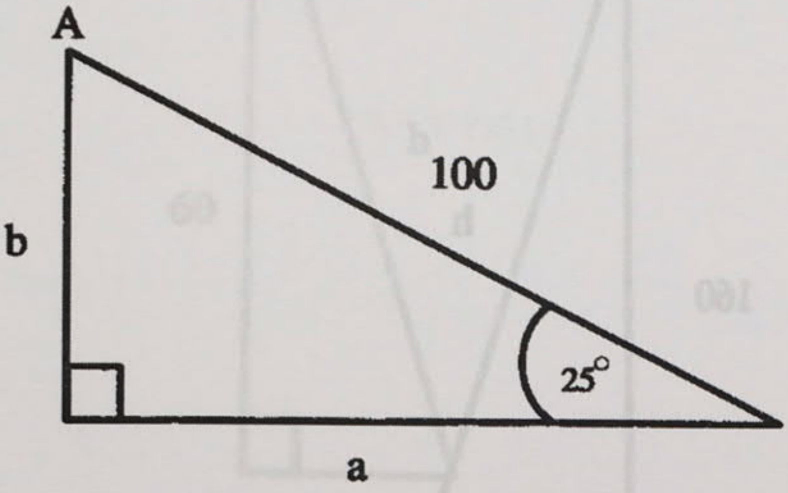
**8.** Find $\sum_{i=1}^{10} (2i+3)$.

**9.** Find $\sum_{i=1}^{\infty} 3\left(\frac{2}{3}\right)^n$.

**10.** Find $\sum_{i=0}^{\infty} \frac{1}{2}\left(\frac{3}{2}\right)^n$.

# *Lesson 30*
# *Introduction to Trigonometry*

Use the figure below to answer questions 1.–4.



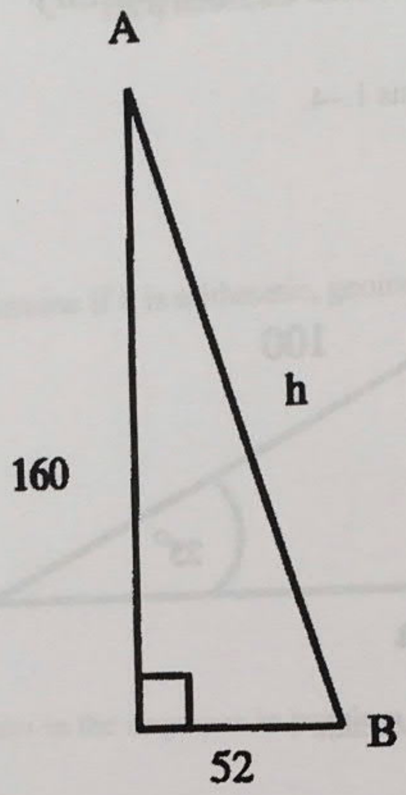
**1.** Find the length of *b* to the nearest tenth.

**2.** Find angle A.

**3.** Find the length of *a* to the nearest tenth.

**4.** Verify that the answers in problems 1 and 2 are correct using the Pythagorean Theorem.

Use the figure below to answer questions 5.–8.



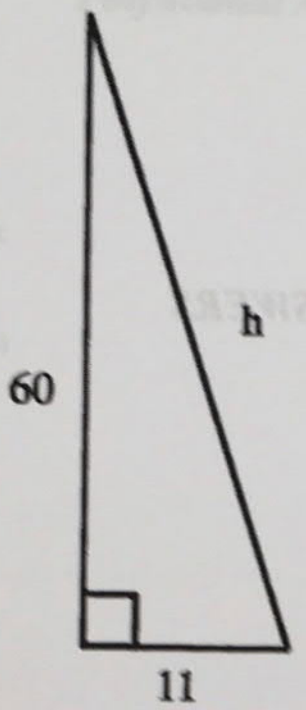
5. Find angle B to the nearest degree.

6. Find angle A.

7. Find side *h* to the nearest tenth.

8. Verify your answer to problem 7 using the Pythagorean Theorem.

Use the figure below to answer questions 9.–10.



9. Use the Pythagorean Theorem to find *h*.

10. A triangle has sides of 10, 20, and 25. Is it a right triangle?

# *ANSWERS*

# Lesson 2
# Polynomial Arithmetic

**1.** $x^3 - x^2 - 8x + 4$

**2.** $x^4 + x^3 + 5x^2 - 10x$

**3.** $4x^3 + 9x^2 - 4x - 14$

**4.** $12x^2 + 43x + 35$

**5.** $4x^2 - 20x + 9$

**6.** $15x^2 - 37x - 8$

**7.** $x^4 + 3x^3 - 2x^2 - 3x + 1$

**8.** $(2x^2 + x - 6) - (6x^2 + 11x - 10)$

$-4x^2 - 10x + 4$

**9.** $(12x^3 - 8x^2 + 28x) - (2x - 1)(2x - 1)$

$(12x^3 - 8x^2 + 28x) - (4x^2 - 4x + 1)$

$12x^3 - 12x^2 + 32x - 1$

**10.**

$$\begin{array}{r} (6x^2 - 11x - 10) \\ x + 7 \\ \hline 42x^2 - 77x - 70 \\ 6x^3 - 11x^2 - 10x \\ \hline \end{array}$$

$6x^3 + 31x^2 - 87x - 70$

# *Lesson 3*
# *Factoring*

**1.** $(2x-1)(4x^2+2x+1)$

**2.** $(2x-5)(3x+7)$

**3.** $(5x-1)(x-5)$

**4.** $3(x^6-64)=$

$3(x^3-8)(x^3-8)(x^3+8)=$

$3(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

**5.** $(x^2+3)(x-4)$

**6.** $3(x^3+x^2-4x-4)=$

$3[x^2(x+1)-4(x+1)]=$

$3(x^2-4)(x+1)=$

$3(x+2)(x-2)(x+1)$

**7.** $$\frac{(x+4)(x-4)}{(x+4)(x+1)}=\frac{x-4}{x+1}$$

**8.** $$\frac{x(x-1)(x^2+x+1)}{x^2(x^2+x+1)}=\frac{x-1}{x}$$

**9.** $$\frac{(x^2+4)(x^2-4)}{(x^2+4)(x^2-5)}=\frac{x^2-4}{x^2-5}$$

**10.** $\frac{(x+8)(x-5)}{(x+2)(x-5)}=\frac{x+8}{x+2}$

# Lesson 4
## Solving Linear Equations

**1.** $x = 4$

**2.** $-4x + 7 = -2x + 17$

$-10 = 2x$

$x = -5$

**3.** $x = -6$

**4.** $x + 2 = 15$

$x = 13$

**5.** $6x - 15 = 6 - 2x + 3$

$8x = 24$

$x = 3$

**6.** $3x - 7 = 2x - 1$

$x = 6$

**7.** $x = -2$

**8.** $x^2 + 2x - 15 = x^2 + x - 6$

$2x - 15 = x - 6$

$x = 9$

**9.** $x = -29$

**10.** $x = -5$

# Lesson 5
## *Solving Linear Inequalities*

**1.** $x > \frac{7}{2}$

**2.** $x < 6$

**3.** $x \ge -3$

**4.** $x > 9$ or $x < 7$

**5.** $5 \le 3x + 1$ $\quad$ $3x + 1 \le 19$
$-6 \le 3x$ $\quad$ $3x \le 18$
$-2 \le x \le 6$

**6.** $x < -5$ or $x > 2$

**7.** $1 \le x \le 3$

**8.** $-1 < x < 4$

**9.** $3x - 4 \le x$ $\quad$ $3x - 4 \ge -x$
$x \le 4$ $\quad$ $4x \ge 4$
$x \le 2$ $\quad$ $x \ge 1$
$1 \le x \le 2$

**10.** There is no answer, because $|x^2 + 3x - 12|$ cannot be $< -2$.

# Lesson 6

## Correlation, Slope, and Intercepts

**1.** We predict that each additional minute of study will raise the grade by .3.

**2.** A student who does not study will probably get a grade of 23.

**3.** $y = .3x + 23$

**4.** This is the amount of studying one would do to predict a zero grade.

**5.** Each additional minute of study would predict a decrease of 2 points in the grade.

**6.** The grade was higher than predicted by the amount of study time.

**7.** The student always did his or her homework or the student is very smart.

# Lesson 7
# The Equation of a Line

**1.** Neither.

**2.** Parallel—they both have a slope of 2.

**3.** Neither—they are the same line.

**4.** Perpendicular—slopes are –3 and $\frac{1}{3}$.

**5.** $y = 3x + b$

$-5 = 3(2) + b \Rightarrow b = -11$

$y = 3x - 11$

**6.** $m = \frac{16}{-8} = -2$

$2 = -2(5) + b \Rightarrow b = 12$

$y = -2x + 12$

**7.** $y = \frac{1}{2}x + 4$

**8.** *Parallel* means slope $= -1$.

$y = -x + 16$

**9.** $m = -\frac{2}{3}$ $\qquad$ $y = -\frac{2}{3}x - 4$

**10.** $y = \frac{7}{3}x + 7$

# Lesson 8
## Graphing Linear Equations

**1.** $y = x + 1$

**2.** $y = -x + 5$

**3.** $y = 5$ [slope = 0]

**4.** $y = -4x$ [$y$-int is (0,0)]

**5.** $y = -3x - 9$

| | | |
|---|---|---|
| **6.** | $y$-int = 10 | $x$-int = 2 |
| **7.** | –6 | –2 |
| **8.** | 4 | –2 |
| **9.** | 7 | 7 |
| **10.** | –3 | 2 |

# Lesson 9
## Graphs of Linear Inequalities

**1.** Yes

**2.** No

**3.** No—point is on the line.

**4.** Yes

**5.** Yes

**6.** No

**7.** Yes—the line is part of the shaded area.

**8.** Yes

**9.** No

**10.** Yes

# Lesson 10
## Solving Systems of Two Linear Equations

**1.** (2,4)

**2.** (–5,–11)

**3.** (1,–1)

**4.**

$$\begin{aligned} x = 5 - 2y \Rightarrow 2(5-2y) - 3y &= -4 \\ 10 - 7y &= -4 \\ -7y &= -14 \\ x = 5 - 2(2) = 1 \Leftarrow y &= 2 \\ &(1,2) \end{aligned}$$

**5.** (3,2)

**6.** (5,–3)

**7.** Infinite number of solutions. They are the same.

**8.**

$$\begin{array}{r} x + y = 7 \\ \underline{2x - y = 2} \\ 3x = 9 \end{array} \qquad x = 3 \qquad y = 4 \qquad (3,4)$$

**9.** $\left(-2, -\frac{2}{3}\right)$

**10.** $(-\frac{3}{4},-2)$

# Lesson 11

## Solving Systems of Two Linear Equations Using Elimination

**1.**

$$\begin{array}{r} x+y=9 \\ \underline{-x-2y=-8} \\ -y=1 \end{array}$$

$y=-1 \Rightarrow x=10$ (10,–1)

**2.**

$$\begin{array}{r} 9x-6y=-6 \\ \underline{2x+6y=-16} \\ 11x=-22 \end{array}$$

$-2+3y=-8 \Leftarrow x=-2$

$3y=-6$

$y=-2$ (–2,–2)

**3.** (1,4)

**4.** (–2,4)

**5.** (3,–1)

**6.** (–2,1)

# Lesson 12
# Solving Systems of Three Linear Equations and Systems of Linear Inequalities

1.

$$\begin{aligned} 4x + 2y - 2z &= 6 \\ \underline{x - 2y - 3z} &\underline{= -6} \\ 5x - 5z &= 0 \\ \underline{-5x - z} &\underline{= 6} \\ -6z &= 6 \\ z &= -1 \end{aligned}$$

$$\begin{aligned} 2x + y - z &= 3 \\ \underline{3x - y + 27} &\underline{= -9} \\ 5x + z &= -6 \end{aligned}$$

$$\begin{aligned} \text{if } z &= -1 \\ 5x + (-1) &= -6 \\ 5x &= -5 \\ x &= -1 \\ 2(-1) + y - (-1) &= 3 \\ y &= 4 \end{aligned}$$

$(-1,4,-1)$

2. $(2,-2,5)$

3. $(7,-2,3)$

4. $(-3,0,1)$

5. $(4,-1,-4)$

6. Yes—on the line is a solution.

7. No

8. No

9. No

10. Yes

# *Lesson 13*
# *Functions*

**1.** Yes

**2.** Yes

**3.** No, $y$ is undefined at $x = 0$.

**4.** No, $y$ is undefined for $-3 \le x < 0$.

**5.** Yes

**6.** Yes

**7.** $f^{-1}(x) = \frac{x-2}{3}$

**8.** $f^{-1}(x) = \sqrt{x+7}$

**9.** $f^{-1}(x) = \frac{1}{2x}$

**10.** $f^{-1}(x) = \sqrt{x^2 - 1}$

# Lesson 14
## Quadratic Functions

**1.** $(3,-25)$

**2.** $(0,-16)$

**3.** $(6,-16)$

**4.** $(8,0)$ and $(-2,0)$

**5.** No, $(-3,11)$ is on the parabola.

**6.** No, $-16 = -16$.

**7.** No, $2 < 20$.

**8.** Yes, $-6 > -7$.

**9.** Yes, $-5 > -7$.

**10.** $(3,4)$

**11.** $(0,-5)$

**12.** $(5,0)$ and $(1,0)$

**13.** Below

# Lesson 15

## Solving Quadratic Functions

**1.** $(x+9)(x-2)=0$

$x=2,-9$

**2.** $(x-4)(x-5)=0$

$x=4,5$

**3.** $2(x+3)(x-3)=0$

$x=3,-3$

**4.** $0=x^2-3x-28$

$0=(x-7)(x+4)$

$x=7,-4$

**5.** $x^2+6x=7$

$x^2+6x+9=16$

$(x+3)^2=16$

$x=-3\pm 4$ or $-7,1$

**6.** $x^2-8x+2=0$

$x^2-8x+16=-2+16$

$(x-4)^2=14$

$x=4\pm\sqrt{14}$

**7.** $\pm\sqrt{12}$ or $\pm 2\sqrt{3}$

**8.** 0

**9.** $\pm\sqrt{2}$

**10.** ±4

# Lesson 16
## The Quadratic Formula

**1.** $a = 1$ $b = -6$ $c = -72$

$b^2 - 4ac = 36 - 4(1)(-72) = 36 + 288 = 324$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{324}}{2} = \frac{6 \pm 18}{2}$$

$x = 12, -6$

**2.** $\frac{-3 \pm \sqrt{41}}{4}$

**3.** $-3$

**4.** $5, -4$

**5.** No real roots

**6.** $\left(-3\frac{1}{4}, -28\frac{1}{8}\right)$

**7.** $(0, -7)$

**8.** $\left(-6\frac{1}{2}, -7\right)$

**9.** $(-7, 0)\left(\frac{1}{2}, 0\right)$

**10.** $-7, \frac{1}{2}$

# *Lesson 17*
# *Imaginary Numbers*

**1.** $3 + 4i$

**2.** 30

**3.** $13 - 11i$

**4.** $(8 - 3i)(8 + 3i) = 73$

**5.** $\frac{2+3i}{3+4i}\frac{(3-4i)}{(3-4i)} = \frac{18+i}{25}$

**6.** $3 + 7i + 13 + 13i = 16 + 20i$

**7.** $-3 + 5i$

**8.** 6

**9.** $7 - \frac{2}{3}i$

**10.** $\frac{2 \pm 8i}{2} = 1 \pm 4i$

**11.** $\frac{-3 \pm i\sqrt{31}}{2}$

**12.** $8+\frac{2}{3}i$

# Lesson 18
## Quadratic and Rational Inequalities

|  |  |  |  |
|---|---|---|---|
| **1.** | $(x+4)(x-3)>0$ |  |  |
|  | $x+4>0, x-3>0$ | OR | $x+4<0, x-3<0$ |
|  | $x>-4, x>3$ |  | $x<-4, x<3$ |
|  | $x>3$ | OR | $x<-4$ |
| **2.** | $x \geq 4$ | OR | $x \leq 2$ |
| **3.** | $x<-5$ | OR | $x>-4$ |
| **4.** | $(x+6)(x-1) \leq 0$ |  |  |
|  | $x \leq -6, x \geq 1$ | OR | $x \geq -6, x \leq 1$ |
|  | [This can't be.] |  | $-6 \leq x \leq 1$ is the answer. |

**5.**

$$x^2-7x+12=0$$

(Alternative method)

$$(x-3)(x-4)=0 \Rightarrow x=3,4$$

When $x$ is 2, $(x-3)(x-4)$ is positive.

| | |
|---|---|
| $3\frac{1}{2}$ | negative |
| 5 | positive |

Therefore, the solution is $3<x<4$.

**6.** $-5 < x < 8$

**7.** $x-3>0, x+4>0$ OR $x-3<0, x+4<0$

$x>3, x>-4$ $\quad$ $x<3, x<-4$

$x>3$ OR $x<-4$

**8.** $x-7 \geq 0, 2x-4>0$ OR $x-7 \leq 0, 2x-4<0$

$x \geq 7, x>2$ OR $x \leq 7, x<2$

$x \geq 7$ OR $x<2$

**Note**: $2x-4$ CANNOT be zero.

**9.** $3x-5 \leq 0, 4-x>0$ OR $3x-5 \geq 0, 4-x<0$

$x \leq \frac{5}{3}, x<4$ $\quad$ $x \geq \frac{5}{3}, x>4$

$x \leq \frac{5}{3}$ OR $x>4$

**10.** $2x+3<0, x+9>0$ OR $2x+3>0, x+9<0$

$x<-\frac{3}{2}, x>-9$ OR $x>-\frac{3}{2}, x<-9$

[This cannot be.]

Solution is $-9<x<-\frac{3}{2}$

# Lesson 19
## Polynomial Division

**1.**

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -2 & 4 \\ & & -1 & -2 & 4 \\ \hline & 1 & 2 & -4 & \underline{8} \end{array}$$

$$x^2 + 2x - 4 + \frac{8}{x+1}$$

**2.**

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -3 & 6 & 16 \\ & & 2 & 4 & 2 & 16 \\ \hline & 1 & 2 & 1 & 8 & \underline{0} \end{array}$$

$$x^3 + 2x^2 + x + 8$$

**3.** $x^4 - 3x^2 + 6x - 16 \div (x-2)(x+2)$

From problem 2, we are left with:

$x^3 + 2x^2 + x + 8 \div x + 2$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 8 \\ & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & \underline{6} \end{array}$$

$$x^2 + 1 + \frac{6}{x+2}$$

**4.** $x^2 - 9x + 27 - \dfrac{1}{x+3}$

**5.** $x^3 + 2x^2 + 4x + 8$

**6.**

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -1 & 0 \\ & & -2 & 4 & -6 \\ \hline & 1 & -2 & 3 & \underline{-6} \end{array}$$

$$x^2 - 2x + 3 - \frac{6}{x+2}$$

**7.** From problem 1, we know that the division problem has a remainder (of 8). Therefore, $x + 1$ is not a factor.

**8.** Yes—from problem 2, because there is no remainder.

**9.** $x - 2$ from problem 5.

**10.** Divide by 2: $\frac{1}{2}x^2 + 2x + 3 \div x + \frac{5}{2}$

$$\begin{array}{r|rrr} -\frac{5}{2} & \frac{1}{2} & 2 & 3 \\ & & -\frac{5}{4} & -\frac{15}{8} \\ \hline & \frac{1}{2} & \frac{3}{4} & \frac{9}{8} \end{array}$$

$$\frac{1}{2}x + \frac{3}{4} + \frac{9}{8\left(x + \frac{5}{2}\right)}$$

$$\frac{1}{2}x + \frac{3}{4} + \frac{9}{8x + 20}$$

# Lesson 20

## Zeros of a Polynomial

**1.** $\pm 1, \pm 2, \pm 4$

**2.** $\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

**3.** By factoring, we find $x^3 + 1 = (x + 1)(x^2 - x + 1)$

$x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

The zeros are $-1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$

**4.**

$$\begin{array}{r|rrrrr}
\underline{2} & 1 & -1 & -7 & 5 & 10 \\
 & & 2 & 2 & -10 & -10 \\
\hline
\underline{-1} & 1 & 1 & -5 & -5 & \underline{|0} \\
 & & -1 & 0 & 5 & \\
\hline
 & 1 & 0 & -5 & \underline{|0} &
\end{array}$$

$$\Rightarrow x^2 - 5 = 0$$

$$x = \pm\sqrt{5}$$

The zeros are $2, -1, \sqrt{5}, -\sqrt{5}$

**5.** 0,3

**6.** $5, i, -i$

**7.** $(-1,0)$ is the only one.

**8.** $(x-2)(x+1)(x^2-5)$ or

$(x-2)(x+1)(x+\sqrt{5})(x-\sqrt{5})$

**9.** (0,0) and (3,0)

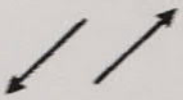
**10.** $(x-5)(x^2+1)$ or

$(x-5)(x+i)(x-i)$

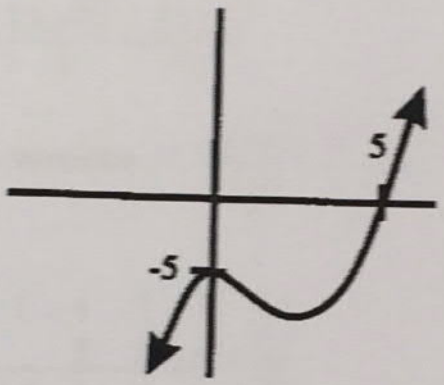
# Lesson 21
# Sketching Polynomials

1. (0,5)

2. (5,0)

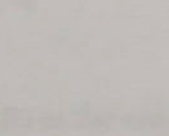
3. 

4. Two or none

5. 

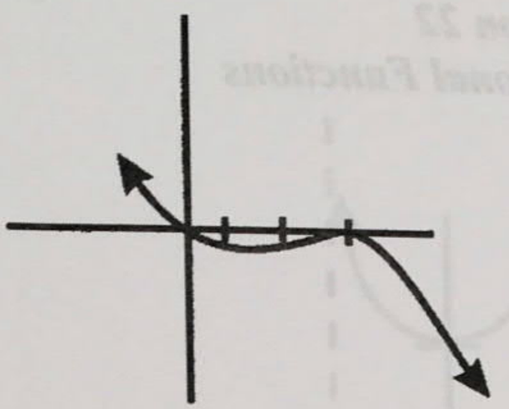
6. (0,0)

7. (0,0) (3,0)

8. 

9. Two or none

**10.**



# Lesson 22
## Sketching Rational Functions

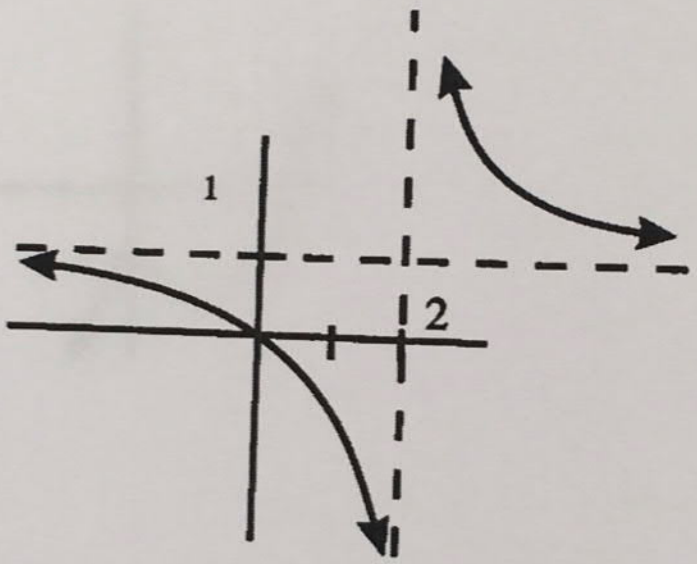
**1.** (0,0)

**2.** (0,0)

**3.** $x = 2$

**4.** $y = 1$

**5.**



**6.** (0,5)

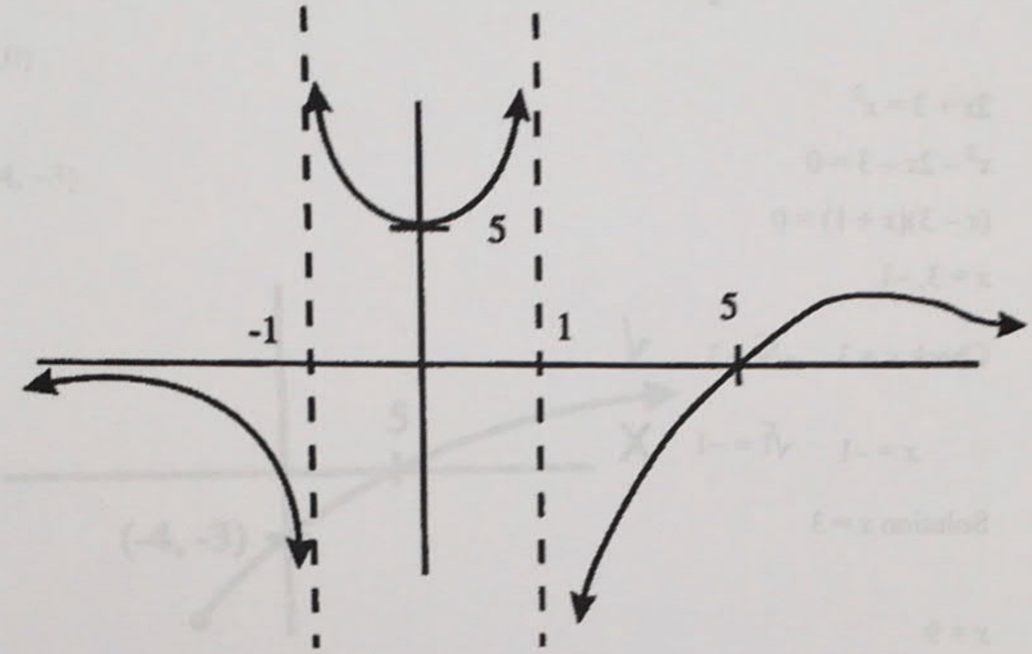
**7.** (5,0)

**8.** $x = 1$

$x = -1$

**9.** $y = 0$

**10.**



# *Lesson 23*
# *Square Roots and Cube Roots*

**1.** $2x + 3 = x^2$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3, -1$

Check $x = 3$ $\sqrt{9} = 3$ ✓

$x = -1$ $\sqrt{1} = -1$ X

Solution $x = 3$

**2.** $x = 9$

**3.** $x = \frac{10}{3}$

**4.** $x + 4 = 8$

$x = 4$

**5.** $x^2 + 2x = x^3$

$x^3 - x^2 - 2x = 0$

$x(x - 2)(x + 1) = 0$

$x = 0,2,-1$

Check $x = 0$ $\sqrt[3]{0} = 0$ ✓

$x = 2$ $\sqrt[3]{8} = 2$ ✓

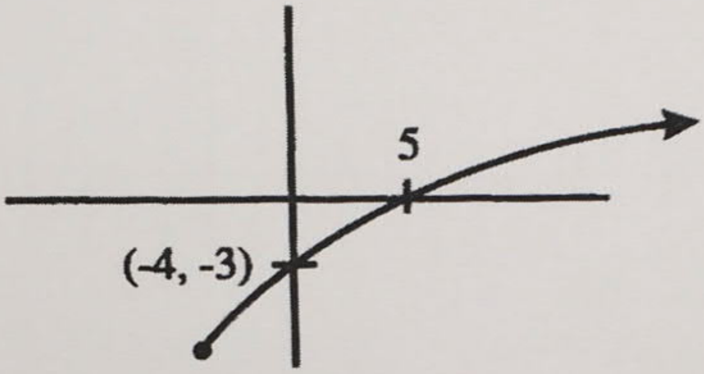
$x = -1$ $\sqrt[3]{-1} = -1$ ✓

**6.** (0,–1)

**7.** (5,0)

**8.** (–4, –3)

**9.**

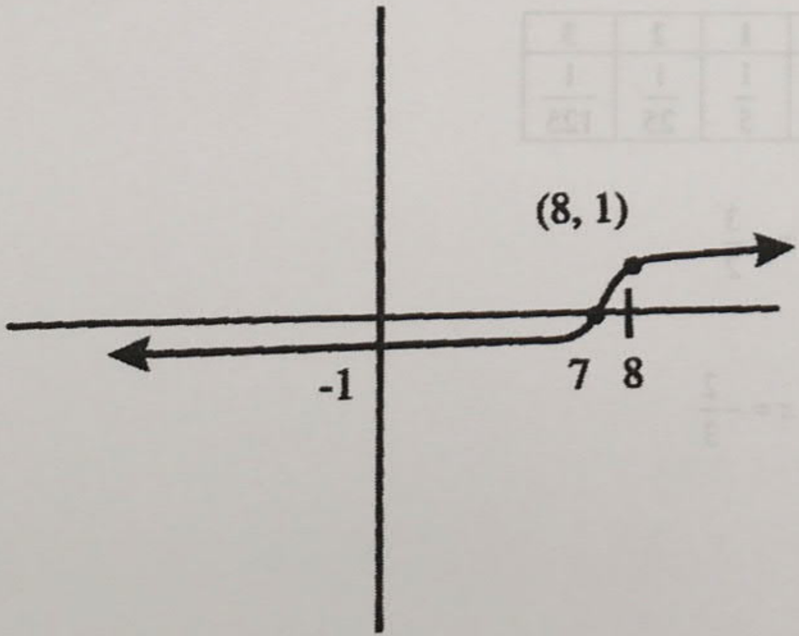


**10.** (0,–1)

**11.** (7,0)

**12.** (8,1)

**13.**



# Lesson 24
## Exponential Functions

**1.** $3^{(2x-2)}$

**2.** $x^{(y-6)}$

**3.** $x^{2y}$

**4.** 1

**5.** $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$

**6.**

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $4$ |
|---|---|---|---|---|---|---|
| $y$ | $\frac{1}{9}$ | $\frac{1}{3}$ | $1$ | $3$ | $9$ | $81$ |

**7.**

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $125$ | $25$ | $5$ | $1$ | $\frac{1}{5}$ | $\frac{1}{25}$ | $\frac{1}{125}$ |

**8.** $3^{2x} = 3^{3}$ $\qquad x = \frac{3}{2}$

**9.** $2^{3x} = 2^{-2}$ $\qquad x = -\frac{2}{3}$

**10.** $2^{2x+2} = 2^{5x}$

$3x = 2$ $\qquad$ $x = \frac{2}{3}$

# Lesson 25
# Logarithmic Functions

**1.** $x^2 = 36$

$x = 6$

**2.** $5^{-3} = x$

$x = \frac{1}{125}$

**3.** $7^x = 343$ $x = 3$

**4.** $x^{\frac{4}{3}} = 16$

$x^{\frac{1}{3}} = 16^{\frac{1}{4}} = 2$

$x = 2^3 = 8$

**5.** $\log(5x) = 2$

$10^2 = 5x$

$x = 20$

$\log_4\left(\frac{x+13}{x-2}\right) = 2$

**6.** $4^2 = 16 = \frac{x+13}{x-2}$

$x = 3$

**7.** $\log_7 19 = \log 19 \div \log 7 = 1.51314...$

**8.** $\frac{1}{2}^{x} = 8 \Rightarrow 2^{-x} = 2^{3} \quad x = -3$

**9.** No, since $\log(-2) = x$, which means that $10^{x} = -2$, and 10 to any power must be a positive number.

Means $10^{x} = -2$ and 10 to any power must be a positive number.

**10.**

| $x$ | $\frac{1}{4}$ | $\frac{1}{2}$ | **1** | **2** | **4** | **8** | **16** |
|---|---|---|---|---|---|---|---|
| $y$ | **−1** | $-\frac{1}{2}$ | **0** | $\frac{1}{2}$ | **1** | $\frac{3}{2}$ | **2** |

# Lesson 26

# Matrices and Determinants

**1.** $$\begin{bmatrix} 3 & -2 & 7 \\ 5 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -12 & 6 & -2 \\ 4 & -10 & -6 \end{bmatrix} = \begin{bmatrix} -9 & 4 & 5 \\ 9 & -10 & -10 \end{bmatrix}$$

**2.** Cannot be added—one matrix is a 2 × 3; the other is a 3 × 2.

**3.** $$\begin{bmatrix} -5 & 8 & 5 \\ 8 & 4 & 4 \end{bmatrix}$$

**4.** $$\begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 10 & 9 \\ -6 & -5 \end{bmatrix}$$

**5.** $-4 - -6 = 2$

**6.** $$2\begin{vmatrix} -1 & 2 \\ -3 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 3 & -3 \end{vmatrix}$$

$2(-1 + 6) - 0 + 3(-3 + 3) = 10$

**7.** $$\frac{1}{\begin{vmatrix} 4 & 3 \\ -2 & -1 \end{vmatrix}} \begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} \\ 1 & 2 \end{bmatrix}$$

**8.** $$\begin{bmatrix} 4 & 3 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The matrix multiplied by its inverse equals the identity matrix.

**9.** There is no inverse since the determinant is zero.

**10.**

$$\left(\begin{bmatrix} -14 & 42 \\ -21 & 7 \end{bmatrix} + \begin{bmatrix} -8 & 14 \\ -6 & -4 \end{bmatrix}\right)^{-1} = \begin{bmatrix} -22 & 56 \\ -27 & 3 \end{bmatrix}^{-1} =$$

$$\frac{1}{1446}\begin{bmatrix} 3 & -56 \\ 27 & -22 \end{bmatrix} = \begin{bmatrix} \frac{1}{482} & -\frac{28}{723} \\ \frac{9}{482} & -\frac{11}{723} \end{bmatrix}$$

# Lesson 27
## Solving Systems of Equations Using Matrices

1. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$

   $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

   $x = 2 \qquad y = 4$

2. $x = -5$

   $y = -11$

3. $x = 1$

   $y = -1$

4. $x = 1$

   $y = 2$

5. $x = 3$

   $y = 2$

6. $x = 5$

   $y = -3$

7. $x = 3$

   $y = 4$

8. $x = -\frac{3}{4}$

$y = -2$

**9.**

$$x = \frac{\begin{vmatrix} -3 & 2 & -1 \\ 20 & -3 & 2 \\ -16 & 1 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 \\ 2 & -3 & 2 \\ 3 & 1 & -4 \end{vmatrix}} = 2$$

$$y = \frac{\begin{vmatrix} 3 & -3 & -1 \\ 2 & 20 & 2 \\ 3 & -16 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 \\ 2 & -3 & 2 \\ 3 & 1 & -4 \end{vmatrix}} = -2$$

$$z = \frac{\begin{vmatrix} 3 & 2 & -3 \\ 2 & -3 & 20 \\ 3 & 1 & -16 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 \\ 2 & -3 & 2 \\ 3 & 1 & -4 \end{vmatrix}} = 5$$

**10.** $x = -3$

$y = 0$

$z = 1$

# Lesson 28
## Recursive Functions

**1.**

| $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(n)$ | 5 | 13 | 29 | 61 | 125 |

**2.**

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(n)$ | 1 | 2 | 2 | 3 | 4 | 5 |

**3.**

| $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(n)$ | 7 | 9 | 11 | 13 | 15 |

**4.** Common difference of 2.

**5.** $\text{Slope} = \dfrac{11-9}{2-1} = \dfrac{2}{1} = 2$

**6.** $m = 2 \qquad b = 7 \qquad \text{since } f(0) = 7$

$f(n) = 2n + 7$

**7.**

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(n)$ | 4 | 8 | 14 | 22 | 32 |

**8.** No—differences are 4, 6, 8, 10.

**9.** Yes—second differences are all 2.

**10.** $ax^2 + bx + c = y$

(1) $a + b + c = 4$

(2) $4a + 2b + c = 8$

(3) $9a + 3b + c = 14$

$$\begin{array}{rl} (2) & 4a + 2b + c = 8 \\ -(1) & -a - b - c = -4 \\ \hline (4) & 3a + b = 4 \end{array} \qquad \begin{array}{rl} (3) & 9a + 3b + c = 14 \\ -(1) & -a - b - c = -4 \\ \hline (5) & 8a + 2b = 10 \\ & 4a + b = 5 \end{array}$$

$$\begin{aligned} 4a + b &= 5 \\ -3a - b &= -4 \\ a &= 1 \\ b &= 1 \\ c &= 2 \end{aligned}$$

$$f(n) = n^2 + n + 2$$

# Lesson 29
## Sequences and Series

**1.** $\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30$

**2.** Arithmetic, $d = 2$

**3.** Geometric, $r = 2$

**4.** Neither

**5.** $a_{50} = 7 + 49(2) = 105$

**6.** $a_{20} = 7 + 19(2) = 45$

Sum = $10(7 + 45) = 520$

**7.** $a_{10} = 7(2)^9 = 7(512) = 3584$

**8.** This is an arithmetic series.

Sum = $5(a_1 + a_{10}) \qquad a_{10} = 23$

Sum = $5(5 + 23) = 140$

**9.** This is an infinite geometric series.

$r = \frac{2}{3}$

$$\text{Sum} = \frac{a}{1-r} = \frac{2}{1-\frac{2}{3}} = \frac{2}{\frac{1}{3}} = 6$$

**10.** This is an infinite geometric series.

$r = \frac{3}{2}$

Since $r > 1$, the sum is infinite.

# Lesson 30
## Introduction to Trigonometry

**1.** $\frac{b}{100} = \sin 25^\circ = .422618$

$b = 42.2618 = 42.3$

**2.** angle A + angle B = 90°

A = 65°

**3.** $\frac{a}{100} = \cos 25^\circ = .906308$

$a = 90.6308 = 90.6$

**4.** $42.3^2 + 90.6^2 = 100^2$

$1789.29 + 8208.36 = 9997.65 \approx 10{,}000$

**5.** $\tan B = \frac{160}{52} = 3.077$

$B = 72^\circ$

**6.** $\mathrm{A} + \mathrm{B} = 90^\circ$

$\mathrm{A} = 18^\circ$

**7.**

$\sin 72^\circ = \frac{160}{h} = .95106$

$h = \frac{160}{.95106} = 168.234 = 168.2$

**8.** $160^2 + 52^2 = 168.2^2$

$25{,}600 + 2704 = 28{,}304 \approx 28{,}291.24$

**9.** $h^2 = 60^2 + 11^2 = 3721$

$h = \sqrt{3721} = 61$

**10.** No, $10^2 + 20^2 = 500$

$25^2 = 625$